Exam Kwantumfysica 2

Date	23 August 2012
Room	5412.0025
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the book or slides, nor other notes or books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	6	2a)	12	3a)	6
1b)	6	2b)	6	3b)	12
1c)	12	2c)	12		
		2d)	6		
		2e)	12		

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) Explain what is a Clebsch-Gordan coefficient.

(b) Use the table below to write down the Clebsch-Gordan decomposition of the state $|j_1, j_2; j, m\rangle = |2, 1; 1, 0\rangle$.

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J J …
	M M …
m_1 m_2	Coefficients
m_1 m_2	
⋮ ⋮	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1 m_2	J M		
	1, +1		
+1/2 +1/2	1		
	1, 0	0, 0	
+1/2 −1/2	1/2	1/2	
−1/2 +1/2	1/2	−1/2	
	1, −1		
−1/2 −1/2	1		

1×1/2

m_1 m_2	J M	
	3/2, +3/2	
+1 +1/2	1	
	3/2, +1/2	1/2, +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3
	3/2, −1/2	1/2, −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3
	3/2, −3/2	
−1 −1/2	1	

2×1

m_1 m_2	J M		
	3, +3		
+2 +1	1		
	3, +2	2, +2	
+2 0	1/3	2/3	
+1 +1	2/3	−1/3	
	3, +1	2, +1	1, +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10
	3, 0	2, 0	1, 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10
	3, −1	2, −1	1, −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5
	3, −2	2, −2	
−1 −1	2/3	1/3	
−2 0	1/3	−2/3	
	3, −3		
−2 −1	1		

1×1

m_1 m_2	J M		
	2, +2		
+1 +1	1		
	2, +1	1, +1	
+1 0	1/2	1/2	
0 +1	1/2	−1/2	
	2, 0	1, 0	0, 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3
	2, −1	1, −1	
0 −1	1/2	1/2	
−1 0	1/2	−1/2	
	2, −2		
−1 −1	1		

2×1/2

m_1 m_2	J M	
	5/2, +5/2	
+2 +1/2	1	
	5/2, +3/2	3/2, +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5
	5/2, +1/2	3/2, +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5
	5/2, −1/2	3/2, −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5
	5/2, −3/2	3/2, −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5
	5/2, −5/2	
−2 −1/2	1	

3/2×1/2

m_1 m_2	J M	
	2, +2	
+3/2 +1/2	1	
	2, +1	1, +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4
	2, 0	1, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2
	2, −1	1, −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4
	2, −2	
−3/2 −1/2	1	

3/2×1

m_1 m_2	J M		
	5/2, +5/2		
+3/2 +1	1		
	5/2, +3/2	3/2, +3/2	
+3/2 0	2/5	3/5	
+1/2 +1	3/5	−2/5	
	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6
	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2
	5/2, −3/2	3/2, −3/2	
−1/2 −1	3/5	2/5	
−3/2 0	2/5	−3/5	
	5/2, −5/2		
−3/2 −1	1		

(c) For an atom in a constant, uniform external magnetic field of intermediate strength, both the Zeeman splitting and the finesplitting need to be taken into account. Explain why the matrix elements $\langle lsjm_j|\mu(\vec{L}+2\vec{S})\cdot\vec{B}+\nu\vec{L}\cdot\vec{S}|l's'j'm_j'\rangle$ must be proportional to $\delta_{ll'}$, $\delta_{ss'}$, and $\delta_{m_jm_j'}$, but not to $\delta_{jj'}$ (μ and ν are prefactors independent of the angular momenta). Also explain why this implies that the states $|lsjm_j\rangle$ do not describe the stationary states.

Exercise 2

Consider a particle with charge q in a one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$. When a weak electric field E is applied, a term $H' = -qEx$ is added to the Hamiltonian.

(a) Find the exact energies of the system.

(b) Explain why in perturbation theory there is no first-order shift in energy due to the addition of H'.

(c) Obtain the tightest upper bound to the ground state energy for a Gaussian trial wave function with an appropriately shifted center. Recall the integrals:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}\,, \quad \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{4\alpha^3}}$$

(d) Give an example of a trial wave function that would give an upper bound to the energy of the first excited state.

(e) Use the WKB method to obtain approximations to the energies of the system.

Exercise 3

Consider the Hamiltonian $H = H_0 + H'$, where the states $\psi_n^{(0)}$ form an orthonormal set of eigenstates of H_0 with energies $E_n^{(0)}$, i.e. $H_0\,\psi_n^{(0)} = E_n^{(0)}\,\psi_n^{(0)}$. H' is a perturbation acting from time $t_0 = 0$.

(a) Show that with the following expansion on the states $\psi_n^{(0)}$

$$\psi(t) = \sum_n c_n(t)\,\psi_n^{(0)}\,e^{-i\,E_n^{(0)}t/\hbar},$$

the coefficients satisfy

$$\dot{c}_m = \frac{1}{i\hbar}\sum_n H'_{mn}\,c_n(t)\,e^{i\,(E_m^{(0)}-E_n^{(0)})t/\hbar},$$

where $H'_{mn} = \langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle$.

(b) Consider the particular case of a two-level system consisting of states a and b, with $H' = \theta(t)e^{-t/\tau}V$, where V is an $\vec{r}$-dependent, t-independent potential and τ is a fixed time scale. Derive in first-order time-dependent perturbation theory the probability that the system is in state b for $t > 0$, assuming it is in state a for $t < 0$.